1 The Monty Hall Problem

The Monty Hall Problem is a classic example of probability. Let's say you are on a game show, and you are presented with 3 doors. Behind 1 is a car, and behind the other 2 are goats. Once you havce picked one of the doors, the host reveals one of the doors that you did not select, and shows it has a goat behind it. Now all that is left is 1 car and 1 goat, behind either your door or the other. The game show now gives you a choice: you can switch to the other door or stay with your original choice. Now here is the big question: Should you stick with your door, switch to the other door, or does it make no difference?

1. Before we begin, make a quick prediction. What do you think you should do?

2. In the beginning, what is the probability of picking the door with a car behind it? (This is the probability of eventually getting a car if you do not switch.)

3. In the beginning, what is the probability of picking the door with a goat behind it? (This is the probability of eventually getting a car if you do switch.)

4. After the game show host opens the door with a goat behind it, what is the probability of having a goat behind your door? A car?

5. Does the the probability of you picking either a car or a goat on the first try change if the game show host reveals one of the unpicked doors?

6. What is the probability that the unopened door you did not pick contains a goat? A car?

7. Now that we've gone over the logic, what should you do?

8. In order to double check our logic above, fill out the table below to see by how much switching doors increases our probability of winning the car. Fill in each blank space with either STAY or SWITCH based on what you should do in the given situations.

	Car Is Behind Door #1	Car Is Behind Door #2	Car Is Behind Door #3
You Pick Door #1			
You Pick Door #2			
You Pick Door #3			

9. Using the information you filled out in the table above, by how much more likely is it to win the car if you switch than if you stay?

2 Monty Hall Bonus Round

It's the bonus round on the Monty Hall show and he decides to change it up a bit, so now instead of only 3 doors, there's 100. 1 still has the car, but the other 99 have goats behind them. Now, should you still switch doors?

1. In the beginning, what is the probability of picking the door with a car behind it? A goat?

2. Now, after the game show host opens the door with a goat behind it, what is the probability of having a goat behind your door? A car?

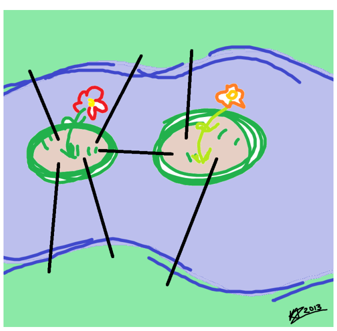
3. What is the probability that the unopened door you did not pick contains a goat? A car?

4. Therefore, what should you do?

3 The Seven Bridges of Königsberg

The Seven Bridges of Königsberg is a historically notable problem in mathematics and was discussed during the 1700s by Leonhard Euler. It eventually laid the foundation for complex math topics like graph theory and topology.

The situation in Königsberg (a city in Prussia at the time) was as follows. A city is split by a river into 2 mainlands and 2 islands, with 7 bridges connecting the land, as shown in the image below. The problem was to devise a walk through the whole city that would cross each of those bridges ONLY one time.



1. First, take a minute to see if you can figure out a method to satisfy the conditions above. Does it matter what shape the rivers are in (i.e. curved, straight, etc.)? What about the bridges?

2. Now, redraw the picture next to the one above, replacing land with *vertices* or dots and replacing rivers with *edges* or lines (they can be curved or straight). The resulting mathematical structure is called a *graph*, often used in graph theory.

3. What do you notice about the number of lines attached to each vertex?

4. Now, if every bridge can only be crossed exactly once, what must be true about the number of bridges touching a certain land mass (or vertex)?

5. Therefore, what can you conclude about this seemingly solvable problem?